GEORGIA INSTITUTE OF TECHNOLOGY
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PROJECT INITIATION

Date November 5, 1963

Project Title: Modes of Vibration of Piezoelectric Quartz Crystals

Project No.: B-251

Project Director: A. L. Bennett

Sponsor: National Science Foundation

Effective: 10-1-63 Estimated to run until: 9-30-65

Type agreement: Grant Award Letter dated 10-24-63 - Grant No. GP-2187

Amount: $61,600.00

Reports: Annual Report
Final Report
Copies of published articles or internal reports

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National Science Foundation
Washington 25, D. C.

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PROJECT TERMINATION

Date June 28, 1967

PROJECT TITLE: Modes of Vibration of Piezoelectric Quartz Crystals

PROJECT NO: B-261

PROJECT DIRECTOR: A. L. Bennett

SPONSOR: National Science Foundation

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ANNUAL REPORT

PROJECT B-261

MODES OF VIBRATION OF PIEZOELECTRIC QUARTZ CRYSTALS

ARTHUR L. BENNETT, PROJECT DIRECTOR
H. GRAHAM HENDERSON, W. F. MARTENS, and SAM A. SCALES

NSF Research Grant GP-2187

1 October 1963 to 30 September 1964

Prepared for
National Science Foundation
Washington, D. C.

1964

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia

GEORGIA INSTITUTE OF TECHNOLOGY
Engineering Experiment Station
Atlanta, Georgia

ANNUAL REPORT

PROJECT B-261

MODES OF VIBRATION OF PIEZOELECTRIC QUARTZ CRYSTALS

By

ARTHUR L. BENNETT, PROJECT DIRECTOR
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I. INTRODUCTION

The purpose of this research is the measurement, analysis, and prediction of the modes of vibration of piezoelectric quartz crystals.

The investigation of the resonances of quartz plates is part of a program at Georgia Tech on frequency control techniques which has been in progress for many years. The work on modes of vibration was begun in 1958 with a systematic measurement of the resonances of a cylindrical disk as the diameter was reduced in small increments. Since the theory of the circular plate has been advanced but little in the intervening years here or elsewhere, attention has since been concentrated on the analysis of rectangular plates.

The theoretical analysis of the modes of vibration of rectangular plates is simpler than for plates with curved boundaries. The responses of a carefully oriented crystal plate of known dimensions were measured carefully for some hundreds of successive decreases of the length by grinding. These spectrum measurements were taken both in the vicinity of the fundamental thickness-shear resonance and of the third harmonic overtone.

In 1963 an approximate theory was published.[1] Appropriate choice of the mathematical model gave eigenfrequencies in good agreement with the measured responses. A second test of the theory is the comparison of the eigenfunctions representing the motion of the plate due to the coupled thickness-shear and flexure modes along the length of the crystal (x-direction). The experimental measurement by the probe technique records the variation of the polarization along a line in the same length. With care, eigenvalues can be chosen which give good agreement between the computed and measured functions. The sensitivity of the computed values to the choice of the eigenfrequency is, however, disturbing.

Further examination of the theory led to the conclusion that additional theoretical investigation was needed, as described in the next section.

II. THEORETICAL

Our efforts have been directed toward the determination of the validity of the approximate theories of Mindlin and his associates and toward the extension of these theories. Familiarization with the programming of numerical computations of the approximations has provided the needed preparation for the solution to the exact problem as proposed by Ekstein.[2] The Mindlin and Gazis approximation[3] serves as the basis of the more sophisticated theories that we have studied. The following brief discussion of the Mindlin and Gazis approximation is intended to present the essential parts of the theory. This outline will be needed for the understanding of further theoretical developments.

A. Mindlin and Gazis Approximate Theory*

Suppose the rectangular plate of thickness 2h is referred to rectangular coordinates, x_i. The x_1 axis coincides with a digonal axis of the quartz, and the x_3 axis makes an angle of $35^{0}15'$ with the trigonal axis. The x_2 axis is chosen to lie in the thickness direction so as to form a right-handed system. The components of displacement are expanded in a power series of the thickness coordinate,

$$u_i = \sum_{j=0}^{N} x_2^{(j)} u_i^{(j)} \qquad (1)$$

where the $u^{(j)}$'s are functions of x_1, x_3, and time, t. This series is truncated for N = 1 when i = 1 or 3, and N = 0 when i = 2. It follows that the differential equations of motion have five dependent variables: three zero order components of displacement, $(u_1^{(0)}, u_2^{(0)}, u_3^{(0)})$, in

*Notation and Development follow Reference 3.

the coordinate directions and two components of rotation, $(u_1^{(1)}, u_3^{(1)})$, of the normal to the plate. We separate the time dependence from the dependent variables by

$$(u_1^{(0)}, u_2^{(0)}, u_3^{(0)}) = (\bar{u}_1, \bar{u}_2, \bar{u}_3)e^{i\omega t}$$

$$(u_1^{(1)}, u_3^{(1)}) = (\psi_1, \psi_3)e^{i\omega t} \tag{2}$$

where ω is the angular frequency, and $\bar{u}_i$ and ψ_i are functions of x_1 and x_3 only.

The fundamental product solutions of the displacement equations given by Mindlin and Gazis are

$$\bar{u}_1 = P_1 \cos (\xi x_1 + \frac{r\pi}{2}) \sin (\zeta x_3 + \frac{s\pi}{2})$$

$$\bar{u}_2 = P_2 \sin (\xi x_1 + \frac{r\pi}{2}) \cos (\zeta x_3 + \frac{s\pi}{2})$$

$$\bar{u}_3 = P_3 \sin (\xi x_1 + \frac{r\pi}{2}) \cos (\zeta x_3 + \frac{s\pi}{2})$$

$$\psi_1 = P_4 \cos (\xi x_1 + \frac{r\pi}{2}) \cos (\zeta x_3 + \frac{s\pi}{2})$$

$$\psi_3 = P_5 \sin (\xi x_1 + \frac{r\pi}{2}) \sin (\zeta x_3 + \frac{s\pi}{2}) \tag{3}$$

where $r = 0$ or 1, $s = 0$ or 1, and where ξ and ζ are the wave numbers in the x_1 and x_3 directions respectively. An infinite series of functions of the type (3) is needed to describe each mode when the boundary conditions are imposed. From this infinite set of functions, Mindlin and Gazis select the following functions

$$\bar{u}_1 = C_1 h \cos\xi x_1 + B_1 h \sin\zeta x_3$$

$$\bar{u}_2 = A_2 h \sin\xi x_1 + D_2 h \cos\zeta x_3$$

$$\bar{u}_3 = A_3 h \sin\xi x_1 + D_3 h \cos\zeta x_3$$

$$\psi_1 = A_4 \cos\xi x_1 + B_4 \cos\zeta x_3$$

$$\psi_3 = C_5 \sin\xi x_1 + D_5 \sin\zeta x_3 \qquad (4)$$

which describe the prominent modes observed experimentally. The boundary conditions are satisfied either on the edge, $x_1 = \pm \ell$, or on the edge, $x_3 = \pm w$, but not on both. This concludes the series of approximations that Mindlin and Gazis find necessary for the solution of the problem.

The solution of the problem is then straightforward. The differentiation operations in the displacement equations of motion may be carried out since we have specified the spatial and time dependence in (4) and (2) respectively. This operation leads to a homogeneous system of linear algebraic equations having as unknowns, the amplitudes (A_2, A_3, A_4), (B_1, B_4), (C_1, C_5), and (D_2, D_3, D_5) which are defined in (4). The equations (4) involving the A's, B's, C's, and D's are not inter-coupled. Therefore for a nontrivial solution, the determinants of the coefficients of the A's, B's, C's, and D's must vanish separately. This condition leads to the characteristic equations relating the wave numbers and the frequency for each of the ten modes of vibration allowed by (4). The solutions for the A's, B's, C's, and D's involve an arbitrary parameter so that only the ratios of the A's, B's, C's, and D's, can be found. Since the four sets of equations are not inter-coupled, one cannot find the ratio of A_4 to B_4, for example. Therefore physical considerations must be used to determine which group of modes of vibration are dominant. An inspection of the piezoelectric relations shows that the component of the electric field in the thickness direction excites only the A and B modes directly.

From (1), (2), and (4), we find that the A modes contribute to the displacement in the following manner

$$
\begin{aligned}
u_1 &= A_4 x_2 \cos \xi x_1 \, e^{i\omega t} + \ldots \text{ (TS}_1\text{ mode)} \\
u_2 &= A_2 h \sin \xi x_1 \, e^{i\omega t} + \ldots \text{ (F}_1\text{ mode)} \\
u_3 &= A_3 h \sin \xi x_1 \, e^{i\omega t} + \ldots \text{ (FS}_1\text{ mode)}
\end{aligned}
\tag{5}
$$

where the TS_1, F_1, and FS_1 represent, respectively, the thickness-shear, flexure, and face-shear modes, all of which are harmonic functions of x_1. Similarly, the B modes contribute to the displacement in the following manner

$$
\begin{aligned}
u_1 &= B_1 h \sin \zeta x_3 \, e^{i\omega t} + \ldots \text{ (FS}_3\text{ mode)} \\
&\quad + B_4 x_2 \cos \zeta x_3 \, e^{i\omega t} + \ldots \text{ (TT}_3\text{ mode)}
\end{aligned}
\tag{6}
$$

where the TT_3 and the FS_3 represent the thickness-twist and face-shear modes, both of which are harmonic functions of x_3.

The application of the boundary conditions leads to a relation between the eigenfrequencies and the length-to-thickness ratio of the crystal plate.

B. Tiersten and Mindlin Theory

The paper by Tiersten and Mindlin[4] presents a refinement of the theory of Mindlin and Gazis, for it includes the piezoelectric effect. The scope of the paper is somewhat more limited because only the TS_1, FS_1, and F_1 modes are considered. The comparison of this theory with experiment will be discussed in part III of this report.

In our experimental work with polarization recordings, we have observed modes of vibration which are harmonic functions of the x_3 coordinate. We know from the Mindlin and Gazis theory that it is possible to solve the vibrational problem involving the TT_3 and FS_3 modes. To the same degree of approximation that was used by Tiersten and Mindlin for the FS_1, F_1, and TS_1 modes, and including the piezoelectric effect, we have found the frequency *vs* length-to-thickness ratio relations necessary for resonance. We are presently making the numerical calculations required for the comparison of the theoretical results with experiment.

C. Coupled x_1 - x_3 Modes of Vibration

In both the Tiersten and Mindlin, and Mindlin and Gazis approximations, a set of displacement functions similar to (4) were selected from fundamental product solutions of the type given by (3). With this selection, the only modes considered are those modes arising from independent plane waves traveling in the x_1 or x_3 directions, as can be seen from examination of the terms in (4). This selection leads to a differential equation eigenvalue problem which is much simpler to solve than the eigenvalue problem associated with the displacement functions given by (3). In this case the eigenvalue problem reduces to two uncoupled eigenvalue problems: one for the plane waves propagating in the x_1 direction, and the second for the plane waves propagating in the x_2 direction. Since these theories do not include the interaction of the x_1 and x_3 modes of neighboring frequencies, they cannot represent exactly the known behavior.

The physical resonances of a bounded plate (both coupled and uncoupled, and dependent oneither x_1 or x_3, or both) will be represented by the solution of the eigenvalue problem associated with the displacement equations (3). The application of the boundary conditions leads to a relation for the relative amplitudes of the different modes and the resonance frequencies in terms of the dimensions of the plate. The resonance frequencies of these coupled modes are determined by both σ_1, and σ_3 (the ratios of the dimension of the plate in the x_1 and x_3 directions to the dimension in the x_2 direction).

D. Comparison With Measurements

One will recall that the Tiersten and Mindlin theory predicted three modes of vibration: face-shear, flexure, and thickness-shear. The application of the boundary conditions leads to a transcendental relation between the frequency of vibration and the dimensions of the plate. The numerical results from this relation were computed on the Burroughs B-5000 computer at Georgia Tech. These results are exhibited in Figure 1 in terms of σ vs Ω, where σ is defined as the length-to-thickness ratio, x_o/y_o', and Ω is defined as the ratio of the frequency of vibration to the frequency of vibration of an infinite plate of the same thickness. The fundamental frequency of the infinite plate, ω_o, is given by

$$\omega_o = \frac{K_6}{\pi y_o'} \sqrt{\frac{3c_{66}}{\rho}} = \frac{1.6561}{y_o'} \text{ Mc/sec}$$

where c_{66} is an elastic stiffness constant, ρ is the density, y_o' is the plate thickness in millimeters, and K_6 is a dimensionless factor. This factor was introduced in the approximate theory by Tiersten and Mindlin to obtain the same fundamental frequency, ω_o, for an unlimited plate as that obtained from the exact theory of an infinite plate.[5]

We have chosen to use the experimentally determined quartz constants of Bechmann[6] and the density of 20° C from Sosman[7], which was confirmed by

Miller and DuMond.[8] Although the constants measured by Koga[9] agree as well as should be expected from the difficulty of the measurement, differences of as much as one per cent introduce a disturbing uncertainty in the eigenvalues computed from either set.

In Figure 2 the computed eigenfrequency curves have been superimposed on the experimental measurements in the vicinity of $\sigma = 12$. The agreement for the curves representing the coupled TS_1 and F_1 modes is good in the flat part of the curves where the fundamental frequency is relatively free of the coupling to the flexure modes. In the steep part of the curves, where the flexure mode predominates, the predicted frequencies fall below the measured ones.

The complete lack of measured responses near the predicted FS_1 curve is unexpected. Japanese measurements, however, show responses with the expected trend. Those measurements were made with plates of larger z_o dimension, $\sigma_z = z_o'/y_o'$ of 36, whereas for our plates, σ_z is 16.4. The indication is that the FS_1 modes are suppressed for plates with small σ_z. This hypothesis will be investigated.

Mindlin and Gazis achieved good agreement with the Japanese experimental data[3] by deriving an empirical set of crystal constants. The same procedure could be used to fit the theory of Tiersten and Mindlin to the Georgia Tech data.

E. Planned Theoretical Work

The mathematical formulation of the Tiersten and Mindlin theory for the x_3 dependent modes will be programmed for the recently acquired Burroughs B-5500 computer. The eigenfrequencies are expected to provide additional identifications for resonances of the crystals measured recently at Georgia Tech and earlier in Japan.

The Tiersten and Mindlin theory has no provision for the interaction of modes propagating in the x_1 and x_3 directions. Both our polarization records and x-ray diffraction topographs* do show a dependence of strain on both x_1 and x_3. An adequate representation of the complex mode interactions will require a more elaborate theory.

The emphasis during the coming period will therefore be on the formulation of an exact theory following Ekstein.[2] Kaul and Mindlin have indicated the approach for an infinite monoclinic plate. Following this lead, the mathematical formulation of the theory of a bounded finite plate will be begun and, we hope, completed during the year. The solution then can be made of the exact theory by successive approximations of the displacement expressions. The programming has become feasible through recent advances at Georgia Tech in this type of functional analysis.

---

*Arthur L. Bennett, R. A. Young, and Norval K. Hearn, Jr., "X-ray Diffraction Topography of Vibrating Quartz Crystals," Applied Physics Letters 2, pp. 154-6, 1963.

III. EXPERIMENTAL

Refinement of measurement techniques has been continued to provide more reliable records for comparison with the theoretical computations.

A. Frequency Spectrum

The spectrum recording equipment, Figure 3, has been improved so that the impedance of each crystal response as well as the frequency can be read from the recording.

The use of the hybrid bridge along with the logarithmic amplifier, as indicated in Figure 3, permits the measurement of the wide variation in impedance exhibited by the quartz crystal while maintaining sufficient accuracy and repeatability for present and planned work. Accuracy of the frequency indication is assured by the use of a stable 100 kc/s quartz oscillator to drive a pulse-shaping circuit and 10 kc/s multi-vibrator. The mixture of the 100 kc/s and 10 kc/s pulses with a sample of the driving signal produces a mark at each 10 kc/s interval of the driving signal with each 100 kc/s interval accentuated. A typical frequency spectrum recorded with the new equipment may be seen in Figure 4.

B. Polarization

The versatility of the polarization recording apparatus has been increased by the construction of a new translator to accomodate larger crystals which have become necessary in testing the theory at increased x_o/y_o' or σ ratio. The detection equipment has been made more accurate and the records easier to interpret by the addition of a phase sensitive detector which determines both the magnitude and sign of the polarization voltage. A block diagram of the components of the recorder is given in Figure 5.

A recording is made by driving the crystal at resonance in one of its modes as indicated by maximum crystal current. The resonator is then traversed mechanically across the probe as shown in Figure 6. The voltage appearing on the probe is recorded as a function of position of the crystal on an x-y recorder. Typical recordings made with the new equipment are shown in Figure 7.

C. Crystal Plates

A set of crystal plates, of which the largest rectangular one is 55 x 55 x 1 mm, has been ordered. The procurement has been difficult because of the stringent requirements on the quality of the natural quartz crystal and the precision of cutting and fabrication. The new plates, both rectangular and circular, will permit full utilization of the improved equipment. The experiments with partial electrodes will also be more significant both with the rectangular and circular crystals.

D. X-Ray Topography

The recent work at Tech has been concentrated on the development of the technique and application to a related project under Army sponsorship. Figures 8 and 9 are topographs of an SL-cut crystal 10 x 4 x 0.4 mm with the length in the x-direction. The SL-cut is a Y-cut rotated CW 52-1/2° about the x axis. The crystal vibrates in the FS_1 mode at a frequency of 455 Kc. Figure 8 is a reflection from the $(2\bar{1}.0)$ plane intersecting the crystal, in the width direction, nearly normal to the plane of the plate. For this reflection, the brightened areas near the longitudinal edge of the crystal correspond to the three nodes of the face shear vibration.

Figure 9 is a reflection from the (01.1) plane intersecting the plate in the length direction. Here the node locations are dark with bright areas between. The combination, in the analysis, of two or more reflections is most helpful in the interpretation of the patterns.

E. Planned Experimental Work

Additional measurements of a variety of plates will be made to determine the effects of source and of fabrication techniques. The new plates will permit measurements at large ratios of length and width to thickness. Our systematic program of frequency measurements will provide more reliable data for comparison with predicted responses.

Polarization measurements continue to be of great assistance in identifying the mode of vibration and the purity of the mode. The new equipment and the larger plates will be used for the extension of the polarization measurements to harmonic resonance regions with meaningful resolution.

Both large and small plates will be used for the measurement of the vibrational patterns resulting from electrodes of reduced size.

X-ray diffraction topographs will continue to be used for analysis of strain patterns. This measurement is independent of the electrical field around the electrodes and will be particularly valuable in the measurement of amplitude distribution around partial electrodes.

IV. PERSONNEL

Dr. D. V. Ho worked one quarter time during the academic year and full time two months during the summer on the applicability of approximate theories.

W. F. Martens, graduate student in math, spent this summer (his second) on the mathematical formulation of approximate theories.

Michael T. Spahr, graduate student in Physics, continued the work on programming the approximate theories for the high speed computers and on comparison of the results with measurements. Personal financial problems led him to discontinue graduate work in March to take industrial employment.

Sam A. Scales, graduate student in Physics, began work in March as an undergraduate. With some assistance from Mr. Spahr, he has taken up the computer programming.

Charles Wagner, graduate student in Physics, started work in the Diffraction Lab in the Fall of 1963. His available time has been limited, but increased participation is expected in the future.

H. Graham Henderson, graduate student in E.E., continued the development of the measurement equipment and the evaluation of the accuracy and significance of the measurements.

Dr. Hughes B. Jenkins, Assistant Professor of Physics at Georgia State, Atlanta, is interested in the theoretical aspects of our work. He is carrying a full schedule of teaching but plans to work on the development of the theory as time permits. In compliance with his desire no charge for his assistance is being made.

V. REFERENCES

1. I. Koga, "Radio-Frequency Vibrations of Rectangular AT-Cut Quartz Plates," Journal of Applied Physics 34, pp. 2357-2365, August 1963.

2. H. Ekstein, "High Frequency Vibrations of Thin Crystal Plates," Physical Review 68, pp. 11-23, 1945.

3. R. D. Mindlin and D. C. Gazis, "Strong Resonances of Rectangular AT-Cut Quartz Plates," Proceedings of the Fourth U. S. National Congress of Applied Mechanics, pp. 305-310, 1962.

4. H. F. Tiersten and R. D. Mindlin, "Forced Vibrations of Piezoelectric Crystal Plates," Quarterly of Applied Mathematics 20, pp. 107-119, 1962.

5. I. Koga, "Thickness Vibrations of Piezoelectric Oscillating Crystals," Physics 3, pp. 70-80, 1932.

6. R. Bechmann, "Elastic and Piezoelectric Constants of Alpha-Quartz," Physical Review 110, pp. 1060-1061, 1958.

7. R. B. Sosman, The Properties of Silica, The Chemical Catalog Company, Inc., pp. 295, 1927.

8. P. H. Miller, Jr. and J. M. DuMond, "Tests for the Validity of the X-Ray Crystal Method for Determining N and e with Aluminum, Silver, and Quartz," Physical Review 57, pp. 198-206, 1939.

9. I. Koga, et al., "Theory of Plane Elastic Waves in a Piezoelectric Crystalline Medium and Determination of Elastic and Piezoelectric Constants of Quartz," Physical Review 109, pp. 1467-1473, 1958.

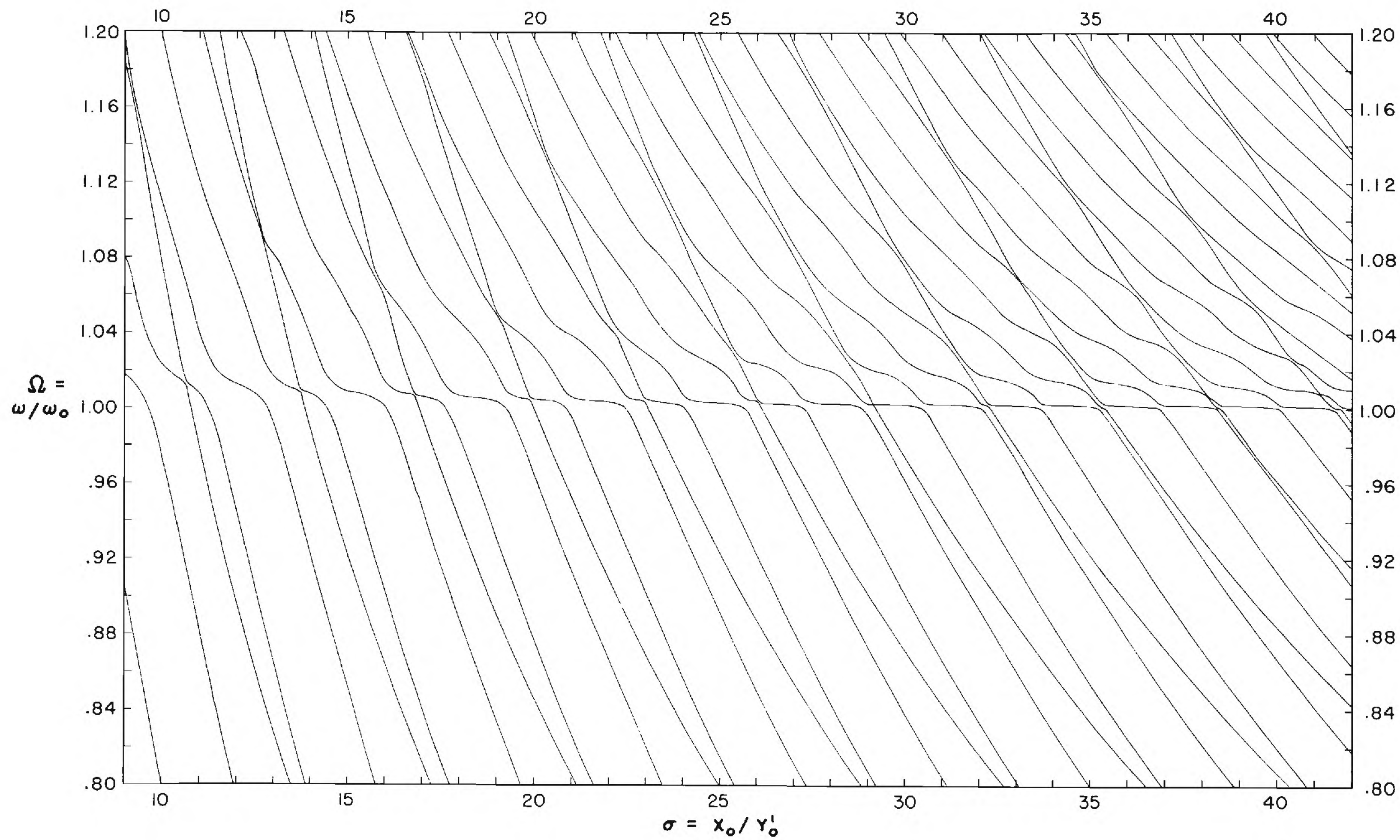


Figure 1. Loci of the resonance frequency eigenvalues, Ω, vs the length-to-thickness ratio, σ. The eigenfrequencies were computed from the approximate theory of Tiersten and Mindlin.

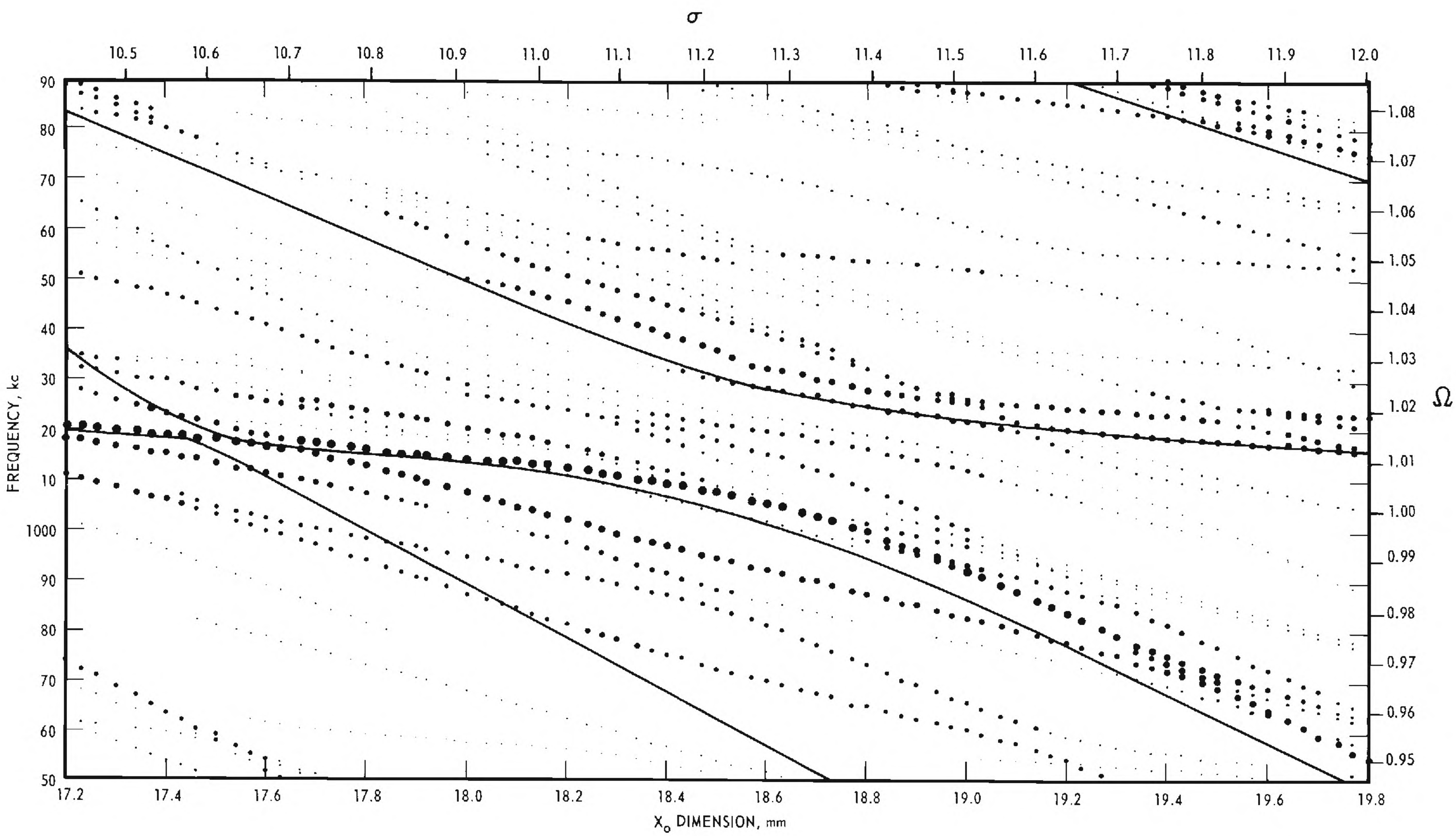


Figure 2. Comparison of the experimentally observed resonances of an AT-Cut quartz plate with the eigenfrequency loci from the Tiersten and Mindlin approximate theory. The plate thickness, y_o, and width, z_o, were 1.650 and 27.004 mm respectively. The length was reduced in small increments as shown by the abscissas. The size of the dot represents the strength of the observed resonance.

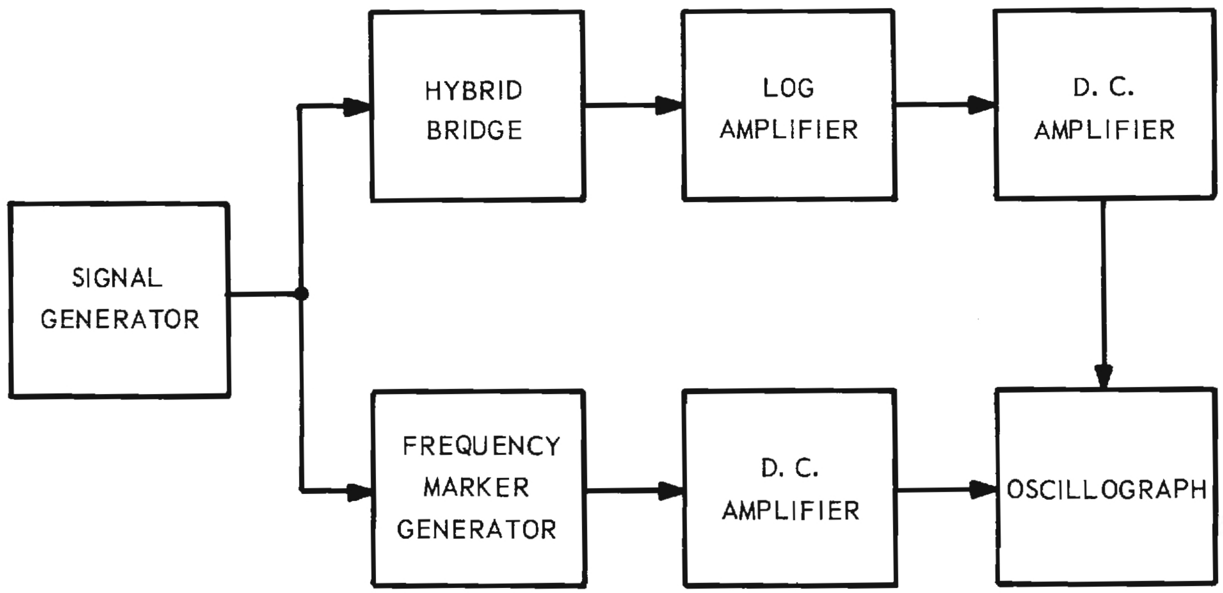


Figure 3. Spectrum recorder diagram.

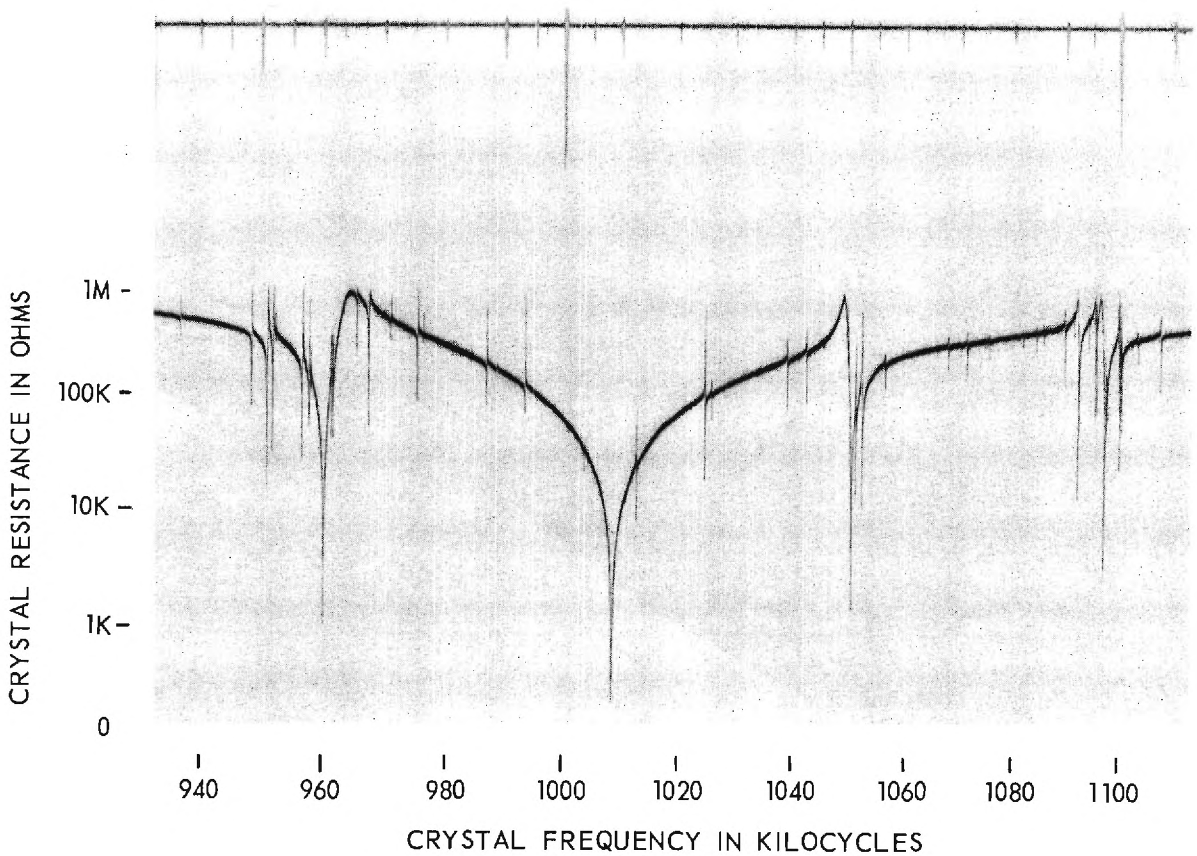


Figure 4. Spectrum of crystal PF.

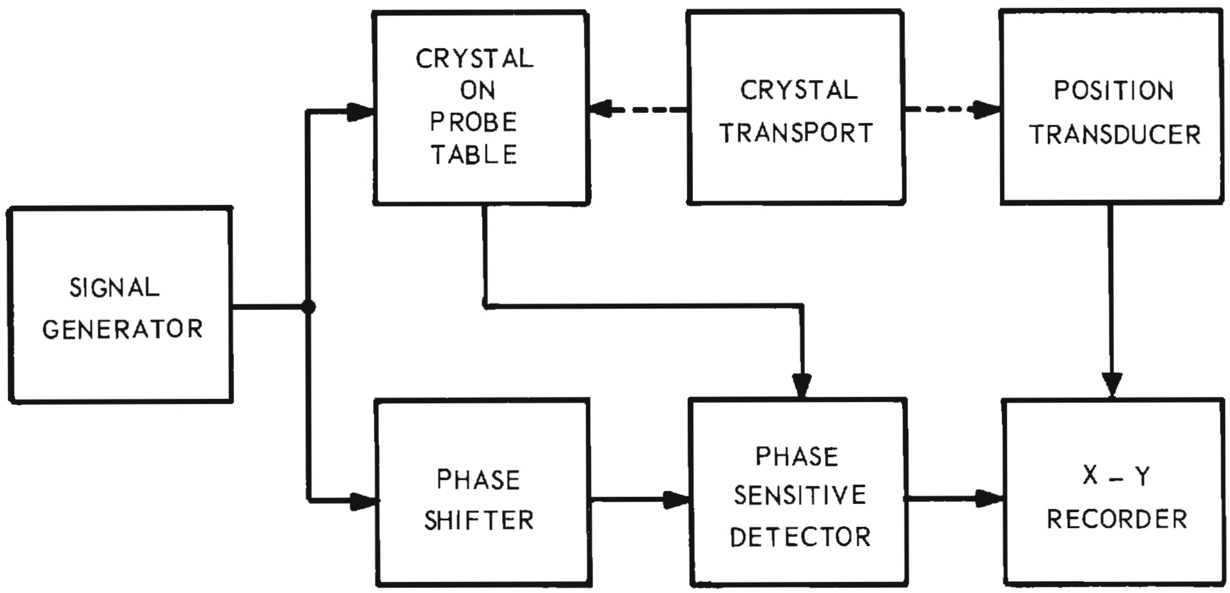


Figure 5. Polarization recorder diagram.

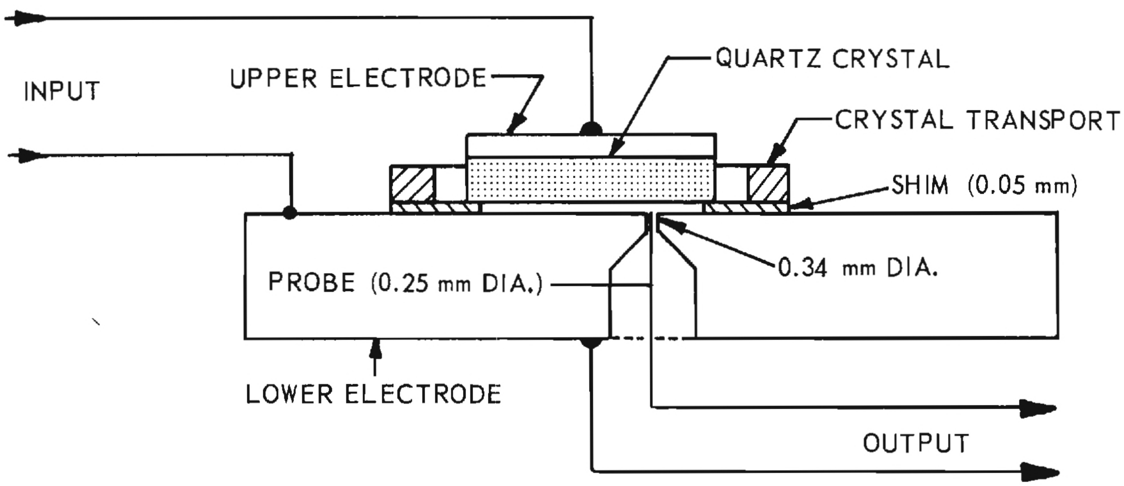
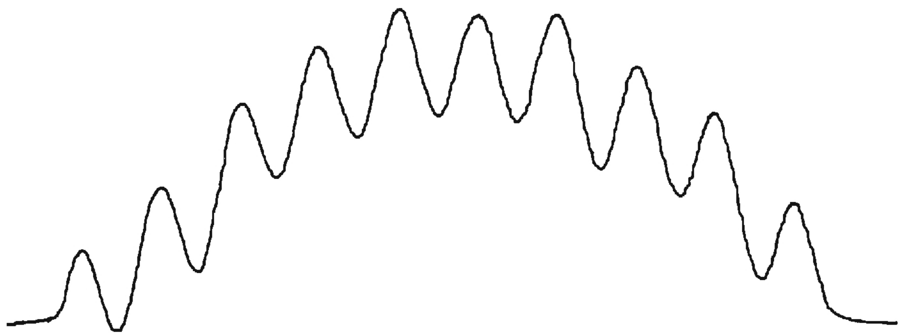
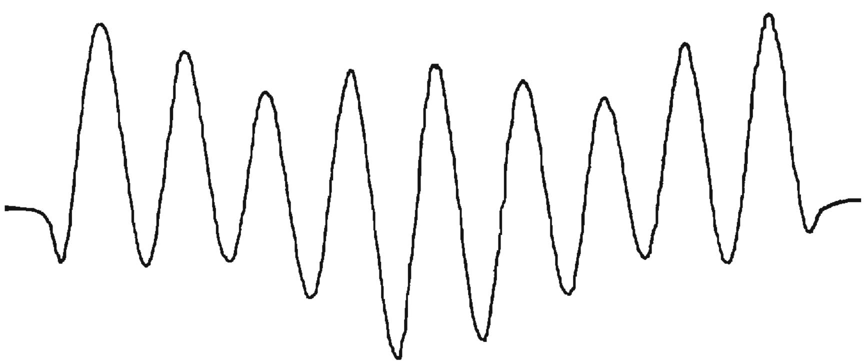


Figure 6. Polarization probe table.



A. FREQUENCY 1008.8 kc



B. FREQUENCY 960.3 kc.

Figure 7. Polarization recordings of two resonances of crystal PF (See Fig. 4). The crystal was traversed in the x direction while centered over the probe in the z' direction.

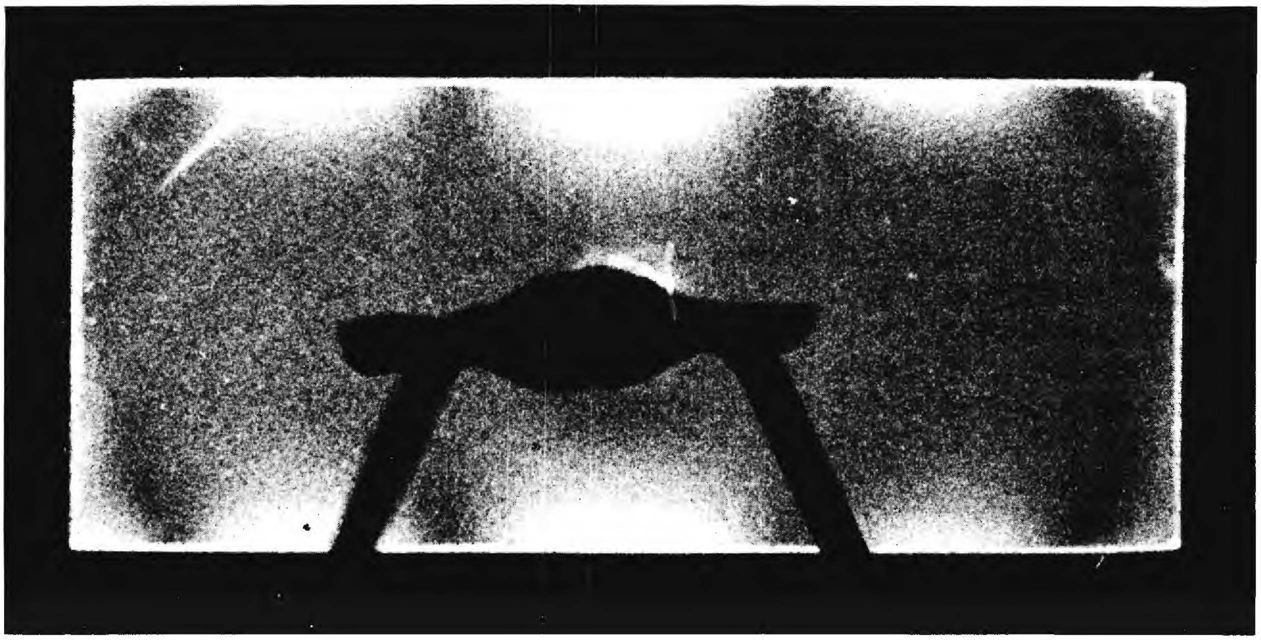


Figure 8. X-ray diffraction topographs of SL-cut quartz plate, $(2\bar{1}.0)$ reflection.

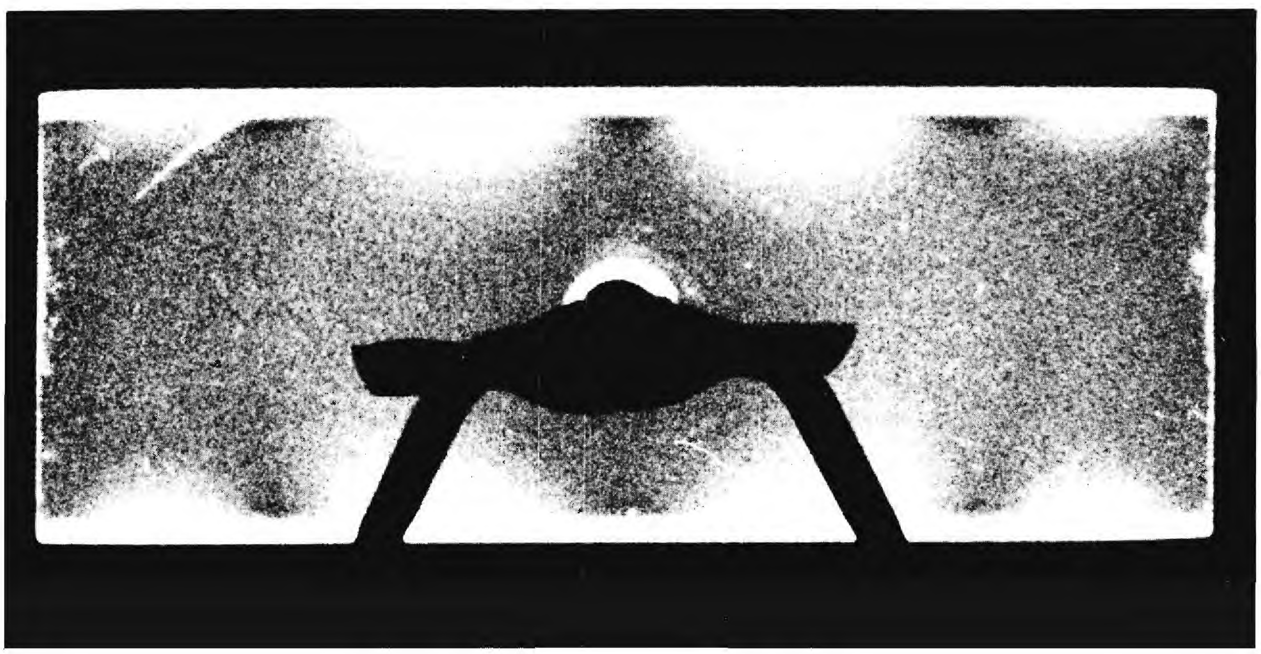


Figure 9. Same plate, (01.1) reflection.

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9 January 1969

The National Science Foundation
Washington, D. C. 20550

Dear Sirs:

This letter is a terminal report on Grant GP-2187. Some work remains to complete the paper which will summarize the work. My apology for the delay in providing this report. The accomplishments during the first year under this grant was reported: Annual Report, Georgia Tech Project B-261.

In continuing the theoretical work, we modified the approach outlined in the Annual Report since no means was found to introduce the width-to-thickness dependence. We then started with the basic formulation by R. D. Mindlin, "High Frequency Vibrations of Crystal Plates," Quarterly of Applied Mathematics 19, 51-61, 1961. The equations of motion (in power series of the displacements) were truncated to reduce the number of dependent variables to five and adjusted to the exact theory for the infinite plate. It was then possible to formulate a computation procedure for the propagation constants in x_1 and x_3. With the introduction of reasonable approximate boundary conditions on both x_1 and x_3, eigenfrequencies representing the four strong coupled modes of vibration could then be computed. This procedure appears to be a solid step forward in the prediction of these frequencies even though the computation procedure is involved and tedious, even on the Burroughs B-5500. Comparison with the experimental data led to an inquiry as to which crystal constants should be used: the best fit was obtained with the accepted elastic stiffness constants (constant field) given by Bechmann and Koga.

Experimental data for comparison with the theory are available in three extensive sets of measurements. The measurements of Hitohiro Fukuyo, Bulletin of the Tokyo Institute of Technology Series A, No. 1, 1955 provide systematic data from three crystals reduced in size in systematic steps with resonances measured at each step. The set designated A was also used by R. D. Mindlin and D. C. Gazis (Proceedings of the Fourth U. S. National Congress of Applied Mechanics, ASME, Vol. I, 305-310, 1962). The data pertains to a crystal of large σ_3 ($z'/y' = 54.0$), held constant, while $\sigma_1 = x/y'$ was reduced from 42 to 34 in 59 steps. The two crystals B and C were of intermediate σ_3, 36.2 and 20.4, with small σ_1, from 9.8 to 4.4.

The measurements made at Georgia Tech, crystal D, provide data at a still smaller σ_3 = 16.4 over the range σ_1 from 14.3 to 9.8.

Comparisons have been made of the predicted resonances with these sets of data. For crystal A, the agreement is as good as found by Mindlin and Gazis; we have, however, used the accepted values of the crystal constants rather than an empirical fit.

The good agreement with the measurements of crystal D with both σ_3 and σ_1 small and for B and C with intermediate values is particularly gratifying.

Much time and energy was spent on the investigation of the new polarization recording equipment. The amplitude and relative strengths of the responses of different modes are strongly dependent on the mounting of the crystal. It is necessary that all surfaces be as free as possible to avoid selective absorption. Consistent results were obtained by attaching a Mylar film 0.003-inch thick to a rectangular frame larger than the crystal. The crystal was placed on the film with an edge parallel to the frame.

Many x-ray topographs of crystals in vibration were made. This technique has the merit that a strain-gradient pattern of all parts of the crystal exposed to the beam are recorded on film. The mechanical requirements of the equipment, however, are even more taxing than that for polarization measurement. With adequate equipment, one can record the strain gradient in the thickness direction, although a correction must be made for the appreciable depth of integration required for reasonable beam intensity.

A description of the technique used for x-ray diffraction topography and illustrations of measurement of AT-cut plates in vibration was published (Appl. Phys. Letters 2, 154, 1963).

Dr. D. V. Ho, School of Mathematics, continued his interest in the project without pay. Fred Martens, Graduate Student in Math, spent a second summer on theoretical work. H. G. Henderson, Graduate Student in E. E., continued with the construction of equipment and in experimental work. Charlie G. Dobson, Jr., Graduate Student in Physics, monitored the computer operation and compared the output with the experimental data. Sam A. Scales, John D. Moses, and Frank Birdsong assisted, both as undergraduates and as graduate students.

The funds provided by this grant have provided research experience to the graduate students indicated above. All have been broadened by the experience as well as stimulated by participating in developing problems.

It is expected that a summary of the results will be published; the support of the National Science Foundation will be acknowledged.

Sincerely yours,

Arthur L. Bennett

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